Transverse observables and the kinematic boundary

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Barr, BMG & Lester, arXiv:0908.3779 [hep-ph]

Outline

- $m_{T(2)}$, kinks &c.
- What does it all mean? The kinematic boundary
- Generalizations: combinatorics, non-identical decays, and the inverse of $m_{T(2)}$

In the beginning ...

The transverse mass

$$m_0^2 = m_v^2 + m_i^2 + 2(E_v E_i - \mathbf{p}_v \cdot \mathbf{p}_i - q_v q_i)$$

- $(E, \mathbf{p}, q)$ is 4-momentum

$$m_T^2 = m_v^2 + m_i^2 + 2(e_v e_i - \mathbf{p}_v \cdot \mathbf{p}_i)$$

- $e = \sqrt{\mathbf{p} \cdot \mathbf{p} + m^2}$ is transverse energy

$$m_0 \geq m_T$$

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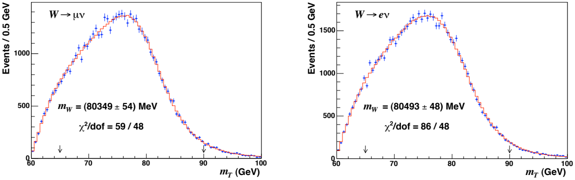
Transverse Mass m_T

$W \to l\nu$

CDF: $m_W = 80.413 \pm 0.048$ GeV

arXiv:0708.3642



Identical Pair Decays and m_{T2}

$$m_T^2 = m_v^2 + m_i^2 + 2(e_v e_i - \mathbf{p}_v . \mathbf{p}_i)$$

- unobservable

$$m_{T2} = \min \max m_T$$

Lester & Summers, PLB 463 99,1999

Barr et al., J.Phys.G29:2343-2363,2003

- observable

$$m_{T2} \leq m_0$$

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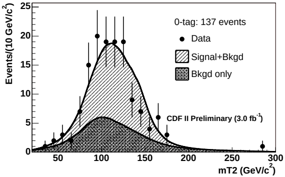
Pair Decays and m_T2

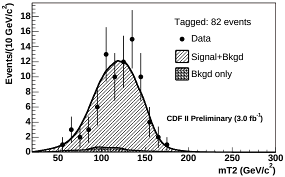
$$t\bar{t} \to 2b2W \to 2b2l2\nu$$

Cho et al. 0804.2185

CDF m_{T2} only: $m_t = 167.9^{+5.6}_{-5.0}$ GeV

CDF note 9769





$m_{T(2)}$ and the kink

$$m_T^2 = m_v^2 + m_i^2 + 2(e_v e_i - \mathbf{p}_v.\mathbf{p}_i)$$

- m_T is unobservable if m_i unknown
- Consider $m_T = m_T(m_i)$
- Lose boundedness but gain a kink

Choi et al., 0709.0288

BMG, JHEP 0208 051 2008

Barr, BMG & Lester, JHEP 0208 014, 2008

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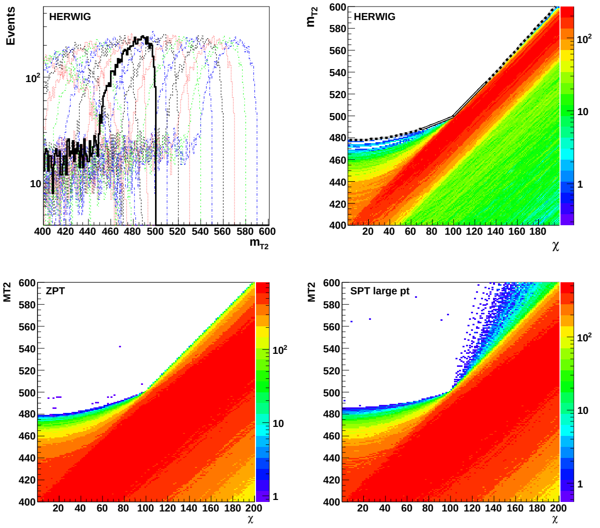
$m_{T(2)}$ and the kink

Pair Three-body decay $2\tilde{g} \to 2q2\overline{q}2\tilde{\chi}_1^0$

Barr, BMG & Lester, JHEP 0208 014,2008



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- *ad hoc* definition of $m_{T(2)}$
- *ad hoc* generalization to hypothesized masses
- In fact these are natural objects ...

Cheng and Han, arXiv:0810.5178

The Kinematic Boundary

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$m_{T(2)}$ is the kinematic boundary of an event

Serna, arXiv:0804.3344

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Kinematic constraints for an event:

- $(p_i + p_\nu)^2 = m_0^2$
- $p_i^2 = m_i^2$
- $\mathbf{p}_i = \not{\mathbf{p}}$

Unknowns are p_i and (m_i, m_0)

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Proof: Two parts

- Part 1: Prove any solution has $m_0 \geq m_T(m_i)$
- Part 2: Prove that there is a solution with $m_0 = m_T(m_i)$

Part 1

Prove any solution has $m_0 \geq m_T(m_i)$

- $m_0^2 = m_i^2 + m_v^2 + 2(E_i E_v - \mathbf{p}_i \cdot \mathbf{p}_v - q_i q_v)$
- $m_T^2 \equiv m_i^2 + m_v^2 + 2(e_i e_v - \mathbf{p}_i \cdot \mathbf{p}_v)$
- $E_i E_v - q_i q_v \geq e_i e_v$, equality at $E_i q_v - E_v q_i = 0$
- $\implies m_0 \geq m_T(m_i)$

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Part 2

Prove $m_0 = m_T(m_i)$ has a solution

- Find suitable E_i, q_i
- Need $\frac{q_i}{q_i^2+e_i^2} = \frac{q_v}{E_v} \in [-1,+1]$

Proof for m_{T2} not much harder

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New definition of $m_{T(2)}$ as "the kinematic boundary of an event"

So what?

- Faster algorithm for computing m_{T2}

  Cheng and Han, arXiv:0810.5178

- $m_{T(2)}$ is the best one can do without extra kinematics or dynamics
- Easier proofs of kinks &c.
- Generalize …

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Generalizations

Generalizations I: Combinatorics

Combinatoric ambiguities

- Branch assignments

Barr and Lester, arXiv:0708.1028

- Initial State Radiation

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- What is the kinematic boundary?
- Union of allowed regions

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Generalizations II: Non-identical decays

Always considered identical pair decays.

But parents/daughters need not be the same

Distinct parents:

- Squark-gluino production in MSSM

Distinct daughters

- Distinct daughters: >1 neutral particle with lifetime $>$ few m
  e.g. Neutrini, Multiple photini

Dimopoulos et al, to appear

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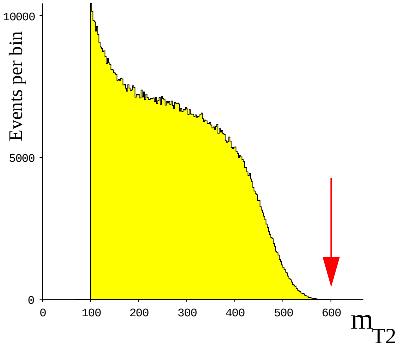
Generalizations II: Non-identical decays

m_{T2} is not much good ...



... use the kinematic boundary?

Generalizations II: Non-identical decays

Focus on case with non-identical parents

- 3-d space of unknown masses (m_i, m_0, m_0')
- Kinematic boundary is a surface

Extremal surface from decoupling argument ...

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Ditto for the other decay, plus

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Last equation decouples for extremal surface

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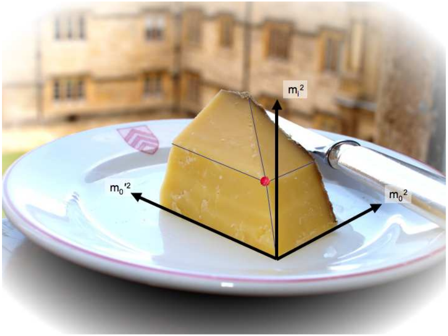
Extremal surface



Lots of interesting kinks . . .

Generalizations II: Non-identical decays

Is there an analogue of the usual m_{T2}?

$$m_{T2}^2(m_i, m_i', \frac{m_0}{m_0'}) = \min\max(\frac{m_0'}{m_0} m_T^2, \frac{m_0}{m_0'} m_T'^2)$$

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Generalizations III: The inverse of $m_{T(2)}$

Use $m_0 = m_{T(2)}(m_i)$ if you know the mass of the daughter but not the parent ...

... what about the other way round?

- Kinematic boundary the same
- Need the inverse of $m_0 = m_{T(2)}(m_i)$

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- $m_T^2 = (\alpha_i + \alpha_v)^2, \ \alpha = (e, \mathbf{p})$
- $(m_T^2)^{-1} = (\alpha_0 - \alpha_v)^2$

- $m_{T2} = \min\max(m_T, m_T')$
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Conclusions

- $m_{T(2)}$ &c are natural objects: define kinematic boundary
- Also the "best" objects
- Easily generalized